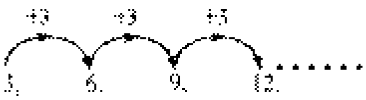
Arithmetic Progressions

- **The Concept of Arithmetic Progression**
  - An arithmetic progression is a list of numbers in which the difference between any two consecutive terms is equal.
  - In an AP, each term, except the first term, is obtained by adding a fixed number called common difference to the preceding term.
  - The common difference of an AP can be positive, negative or zero.

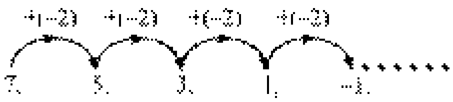
Example 1:

1.

1. 1. 

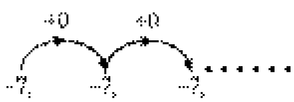
is an AP whose first term and common difference are 3 and 3 respectively.

2.

3. 2. 

is an AP whose first term and common difference are 7 and –2 respectively.

4.

5. 3. 

is an AP whose first term and common difference are –7 and 0 respectively.

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  - The general form of an AP can be written as a, $a + d$, $a + 2d$, $a + 3d$ …, where a is the first term and d is the common difference.
  - A given list of numbers i.e., a_1, a_2, a_3 … forms an AP if $a_{k+1} - a_k$ is the same for all values of k.

Example 2:

Which of the following lists of numbers forms an AP? If it forms an AP, then write its next three terms.

(a) –4, 0, 4, 8, …

(b) 2, 4, 8, 16, …

Solution:

(a) –4, 0, 4, 8, …

$a_2 - a_1 = 0 - (-4) = 4$

$a_3 - a_2 = 4 - 0 = 4$

$a_4 - a_3 = 8 - 4 = 4$

$a_{n+1} - a_n = 4$; for all values of n

Therefore, the given list of numbers forms an AP with 4 being its common difference.

The next three terms of the AP are 8 + 4 = 12, 12 + 4 = 16, 16 + 4 = 20

Hence, AP: –4, 0, 4, 8, 12, 16, 20 …

(b) 2, 4, 8, 16, …

$a_2 - a_1 = 4 - 2 = 2$

$a_3 - a_2 = 8 - 4 = 4$

$a_3 - a_2 \neq a_2 - a_1$

Therefore, the given list of numbers does not form an AP.

- **The terminology related to arithmetic progression**
  - An arithmetic progression is a list of numbers in which each term is obtained by adding a fixed number to the preceding term except the first term.
  - The fixed number is called the common difference (d) of the A.P. The common difference can be either positive or negative or zero.
- **The general form of an A.P.**
  - a, $(a + d)$, $(a + 2d)$, $(a + 3d)$, ... , $[a + (n - 1)d]$, ... where *a is the* first term and *d is* common difference
- **Type of AP**
  - Finite AP: The APs have finite number of terms.
  - Infinite AP: The APs have not finite number of terms.

- In an A.P., except the first term, all the terms can be obtained by adding the common difference to the previous term.
- In an A.P., except the last term, all the terms can be obtained by subtracting the common difference from its subsequent term.

Example:
Find the first four terms of an A.P. whose first term is 9 and the common difference is 6.
Solution:
$a = 9$, $d = 6$
$a_2 = a + d = 9 + 6 = 15$
$a_3 = a + 2d = 9 + 2 \times 6 = 9 + 12 = 21$
$a_4 = a + 3d = 9 + 3 \times 6 = 9 + 18 = 27$
The first four terms are 9, 15, 21, 27.

- **n^{th} term of an AP**

  The n^{th} term (a_n) of an AP with first term a and common difference d is given by $a_n = a + (n - 1)\,d$.

  Here, a_n is called the general term of the AP.
- **n^{th} term from the end of an AP**

  The n^{th} term from the end of an AP with last term l and common difference d is given by $l - (n - 1)\,d$.

Example:
Find the 12th term of the AP 5, 9, 13 …
Solution:
Here, $a = 5$, $d = 9 – 5 = 4$, $n = 12$

$$a_{12} = a + (n – 1)\,d$$
$$= 5 + (12 – 1)\,4$$
$$= 5 + 11 \times 4$$
$$= 5 + 44$$
$$= 49$$

- **Sum of *n* terms of an AP**
  - The sum of the first *n* terms of an AP is given by Sn=n22a+n-1d , where *a* is the first term and *d* is the common difference.

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  - If there are only *n* terms in an AP, then Sn=n2a+l, where $l = a_n$ is the last term.

Example :
Find the value of 2 + 10 + 18 + …. + 802.
Solution:

2, 10, 18… 802 is an AP where $a = 2$, $d = 8$, and $l = 802$.

Let there be *n* terms in the series. Then,
$a_n = 802$
$\Rightarrow a + (n – 1)\,d = 802$
$\Rightarrow 2 + (n – 1)\,8 = 802$
$\Rightarrow 8(n – 1) = 800$
$\Rightarrow n – 1 = 100$
$\Rightarrow n = 101$
Thus, required sum = n2a+l = 10122+802 = 40602

- **Properties of an Arithmetic progression**
  - If a constant is added or subtracted or multiplied to each term of an A.P. then the resulting sequence is also an A.P.
  - If each term of an A.P. is divided by a non-zero constant then the resulting sequence is also an A.P.

- **Arithmetic mean**
  - For any two numbers *a* and *b*, we can insert a number A between them such that *a*, A, *b* is an A.P. Such a number i.e., A is called the arithmetic mean (A.M) of numbers *a* and *b* and it is given by $A = \frac{a+b}{2}$.

- For any two given numbers *a* and *b*, we can insert as many numbers between them as we want such that the resulting sequence becomes an A.P.

Let $A_1, A_2 \dots A_n$ be *n* numbers between *a* and *b* such that $a, A_1, A_2 \dots A_n, b$ is an A.P.

Here, common difference (*d*) is given by $\frac{b-a}{n+1}$.

Example:
Insert three numbers between –2 and 18 such that the resulting sequence is an A.P.

Solution:
Let A_1, A_2, and A_3 be three numbers between – 2 and 18 such that –2, A_1, A_2, A_3, 18 are in an A.P.

Here, $a = -2$, $b = 18$, $n = 5$

$\therefore 18 = -2 + (5 - 1)\, d$

$\Rightarrow 20 = 4\, d$

$\Rightarrow d = 5$

Thus, $A_1 = a + d = -2 + 5 = 3$

$A_2 = a + 2d = -2 + 10 = 8$

$A_3 = a + 3d = -2 + 15 = 13$

Hence, the required three numbers between –2 and 18 are 3, 8, and 13.